A Strategy for Parallel Simulation of Declarative Object-Oriented Models of Generalized Physical Networks

Francesco Casella

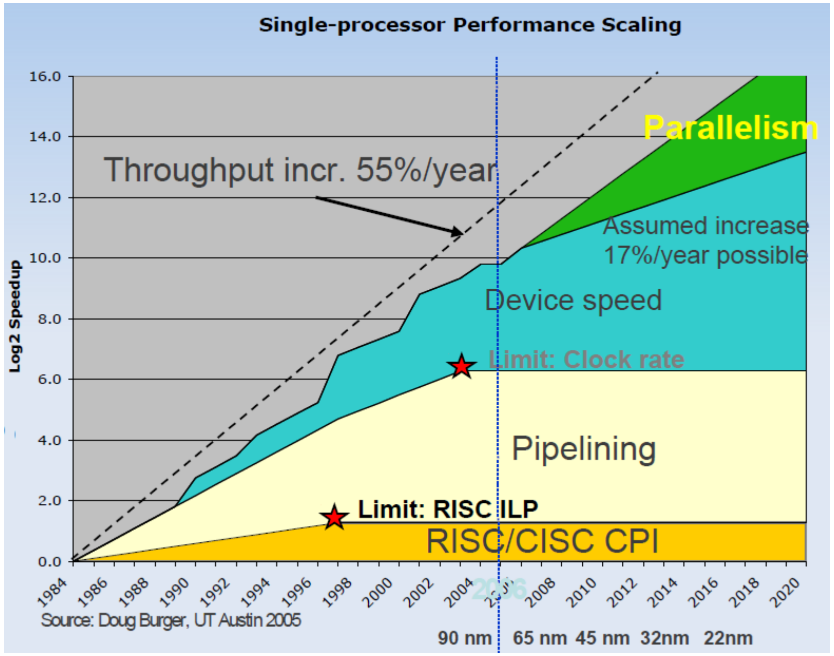
(francesco.casella@polimi.it)

Dipartimento di Elettronica, Informazione e Bioingegneria

Politecnico di Milano

Introduction and motivation

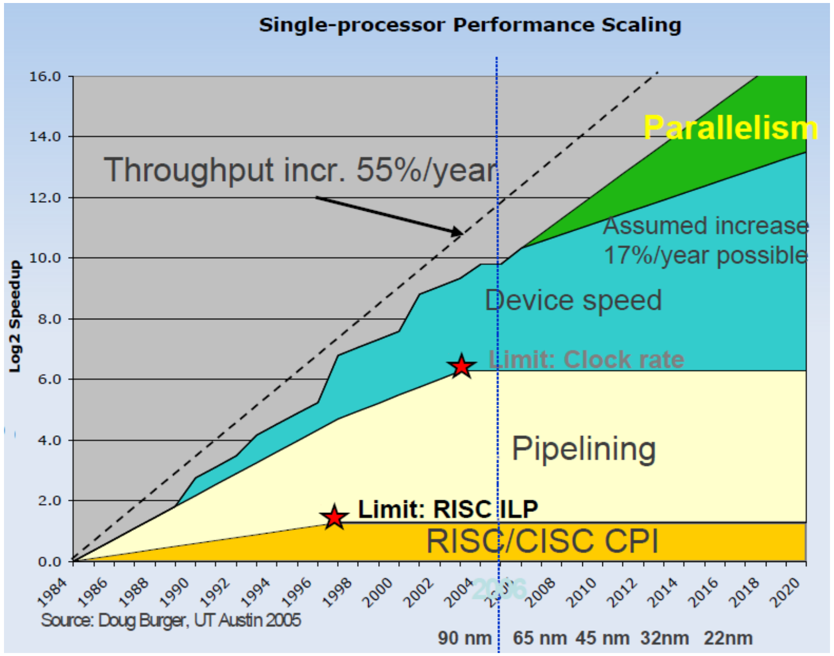
- Moore's law depends on multi-core architectures since 2007
- Declarative O-O modelling is now a mature & established field (1997: Modelica 1.0)



Introduction and motivation

- Moore's law depends on multi-core architectures since 2007
- Declarative O-O modelling is now a mature & established field (1997: Modelica 1.0)

however

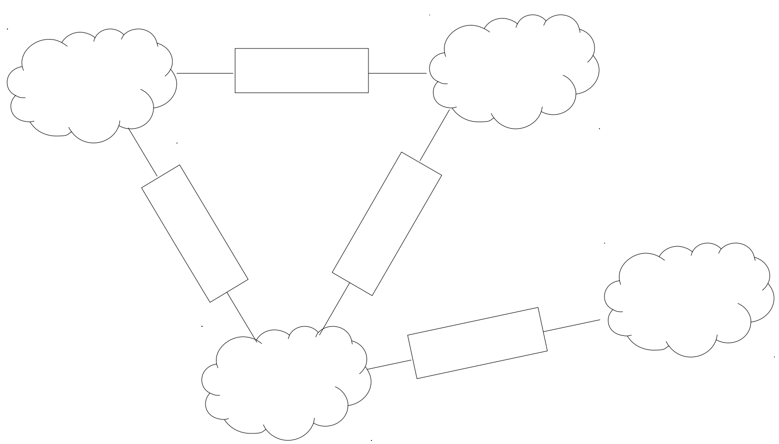


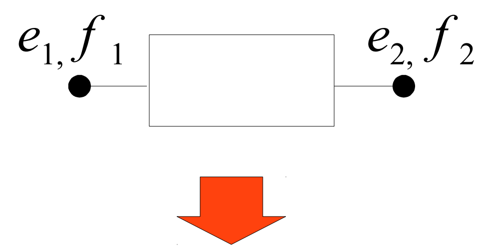
- Simulation code generated by state-of-the-art Modelica tools as of 2012 is still fully sequential!

why?

TLM Modelling

- Main idea: physical interactions given by wave propagation phenomena with finite delay
    - pressure / flow waves in hydraulic circuits
    - electromagnetic waves in transmission lines
    - elastic waves in mechanical systems
- Explicitly partion the system with Transmission Line Models

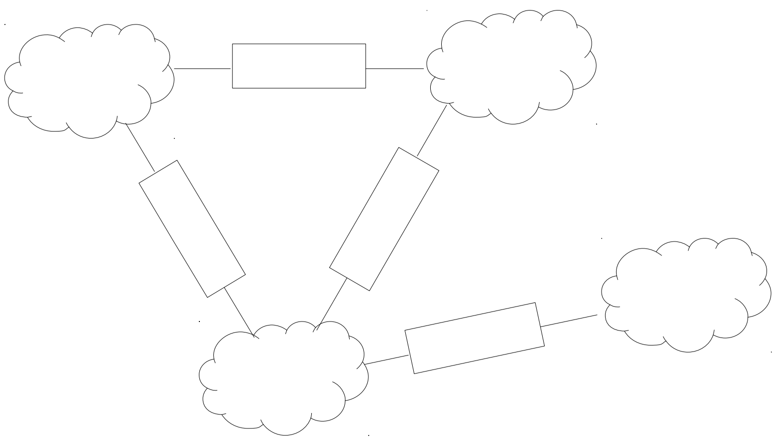


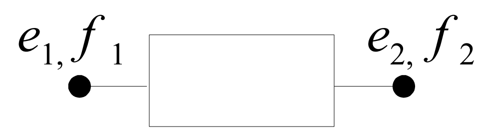


$$e_1(t) = Zf_1(t) + e_2(t - T_{tl}) + Zf_2(t - T_{tl})$$
$$e_2(t) = Zf_2(t) + e_1(t - T_{tl}) + Zf_1(t - T_{tl})$$

TLM Modelling

- Main idea: physical interactions given by wave propagation phenomena with finite delay
  - pressure / flow waves in hydraulic circuits
  - electromagnetic waves in transmission lines
  - elastic waves in mechanical systems
- Explicitly partion the system with Transmission Line Models





$$e_1(t) = Zf_1(t) + e_2(t - T_{tl}) + Zf_2(t - T_{tl})$$
$$e_2(t) = Zf_2(t) + e_1(t - T_{tl}) + Zf_1(t - T_{tl})$$

Sub-systems can be solved independently ($\rightarrow$ in parallel) for T seconds

TLM Modelling

Advantages

- Modelling is physically accurate – no approximations

Disadvantages

- Decoupling elements must be inserted manually
- Physical delays T are usually very small
- Maximum step size constrained by T

➡ requires expertise

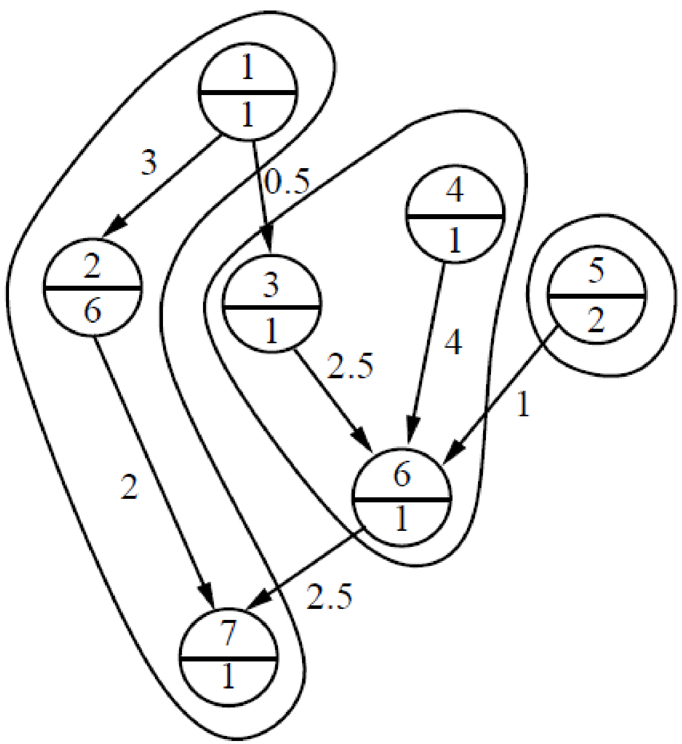
➡ parallel simulation might be slower than sequential simulation of the coupled model

Peter Aronsson's PhD work

$$F(x, \dot{x}, v, t) = 0$$

$$\dot{x} = f(x, t)$$
$$v = g(x, t)$$

explicit assignments
+
implicit equations to solve



Peter Aronsson's PhD work

Advantages

- Applies to any equation-based model without manual intervention
- Optimal scheduling possible in principle

Disadvantages

- Accurate estimation of computation and communication delays crucial
- Potentially bad performance if delays not correct
- Task merging and clustering algorithms very involved
- Possibly too complex for large systems

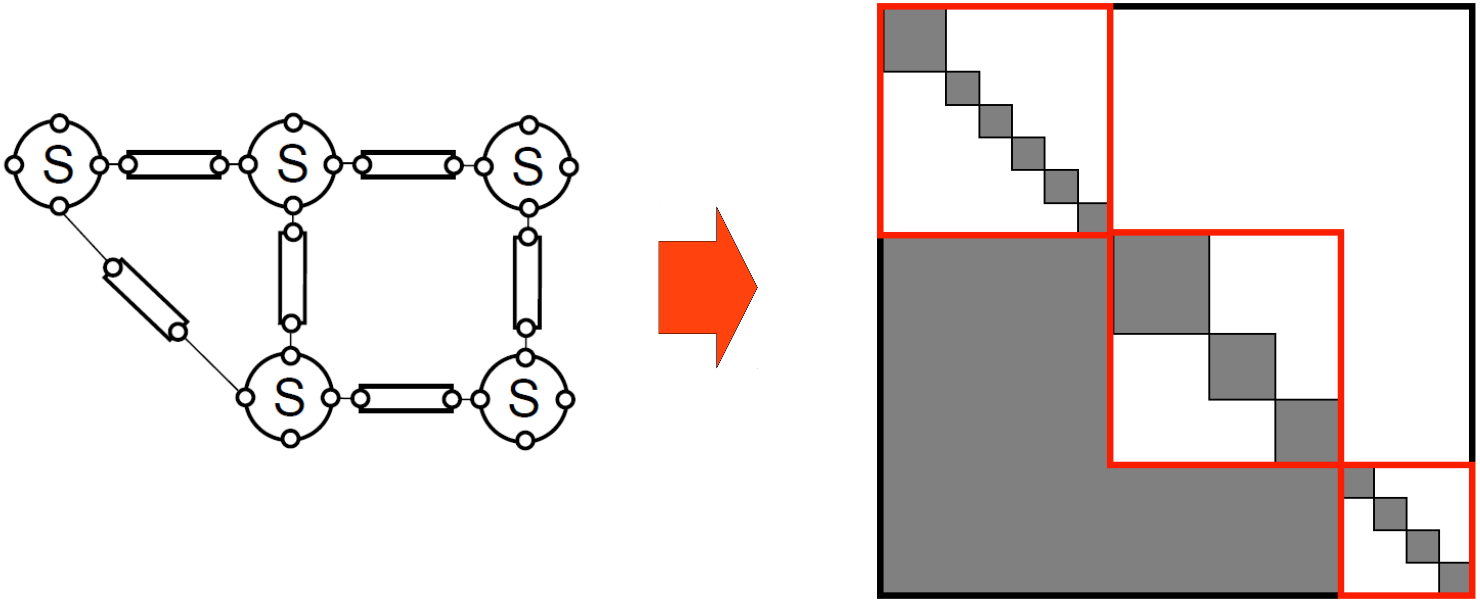
➡ never found way into production tools

Goals of this work

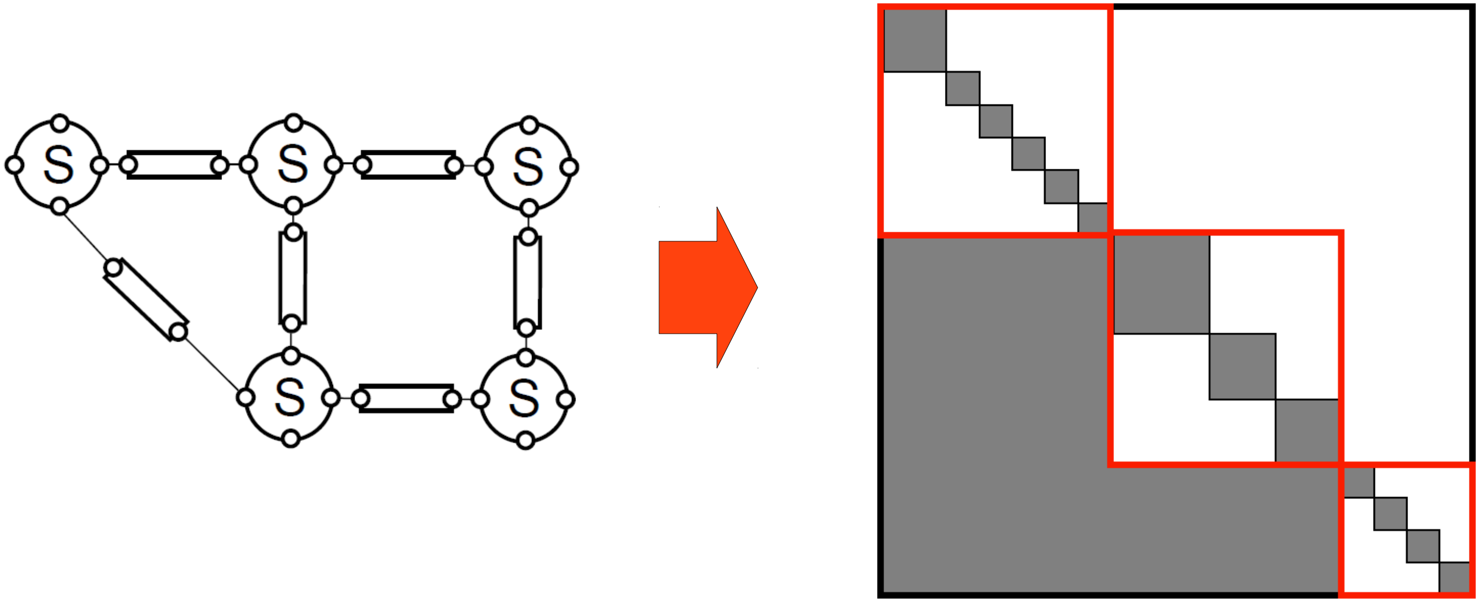
- Identify a (large) class of O-O models that have a special structure of the incidence matrix

- Propose an algorithm which is simple to implement, but provides near optimal performance for that class of models

Main Idea



Main Idea



Exploit for parallelization!

Proposed algorithm

1. Build E-V digraph
2. Find a complete matching
   (possibly using Pantelides / Dummy Derivatives for higher index systems)
3. Replace non-matching edges with E$\rightarrow$V arc; collapse V-nodes with matching E-nodes (a directed graph is obtained)
4. Run Tarjan's algorithm and identify strong components
   (systems of equations to be solved simultaneously)

Proposed algorithm

1. Build E-V digraph
2. Find a complete matching
   (possibly using Pantelides / Dummy Derivatives for higher index systems)
3. Replace non-matching edges with E$\rightarrow$V arc; collapse V-nodes with matching E-nodes (a directed graph is obtained)
4. Run Tarjan's algorithm and identify strong components
   (systems of equations to be solved simultaneously)
5. Collapse each strong component into a single macro-node
6. Let i = 1
7. Search for all sinks and collect them in set S_i
   (they correspond to equations that can be solved independently)
8. Delete all nodes in set S_i and all associated arcs from the graph
9. If there are nodes left, increase i by 1 and goto 6.

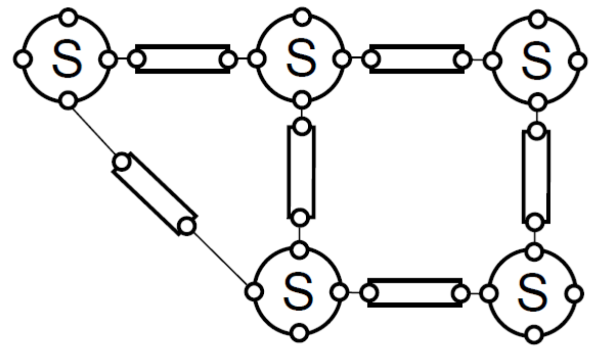
Proposed algorithm

1. Build E-V digraph
2. Find a complete matching
   (possibly using Pantelides / Dummy Derivatives for higher index systems)
3. Replace non-matching edges with E$\rightarrow$V arc; collapse V-nodes with matching E-nodes (a directed graph is obtained)
4. Run Tarjan's algorithm and identify strong components
   (systems of equations to be solved simultaneously)
5. Collapse each strong component into a single macro-node
6. Let i = 1
7. Search for all sinks and collect them in set S_i
   (they correspond to equations that can be solved independently)
8. Delete all nodes in set S_i and all associated arcs from the graph
9. If there are nodes left, increase i by 1 and goto 6.

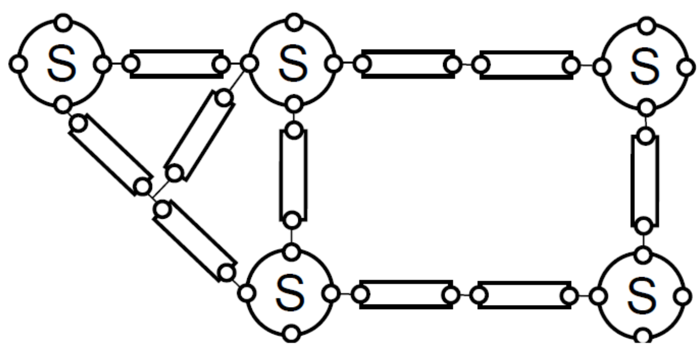
At termination, all equations are collected in the sets S_i
Equations in S_1 can be solved independently,
then equations in S_2, etc.

Generalized networks

- Storage components: storage of physical quantities, represented by state variables (known at each time step!)
- Flow components: describe the flow of the quantity, based on the boundary values

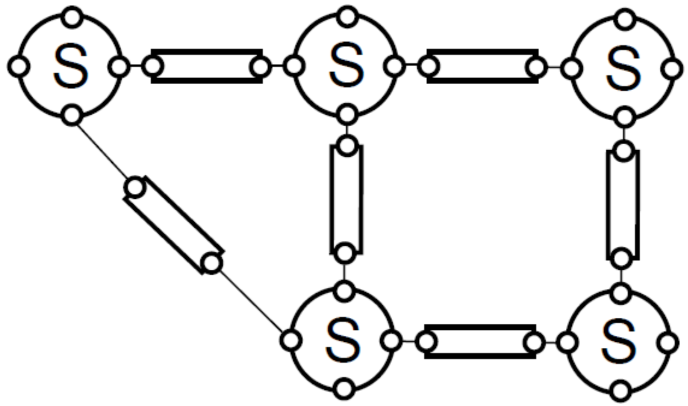


- V-F-V topology:
no implicit equations



- V-F-F-V topology:
implicit equations
(macro-nodes)

Example 1: Thermal Networks



Storage components

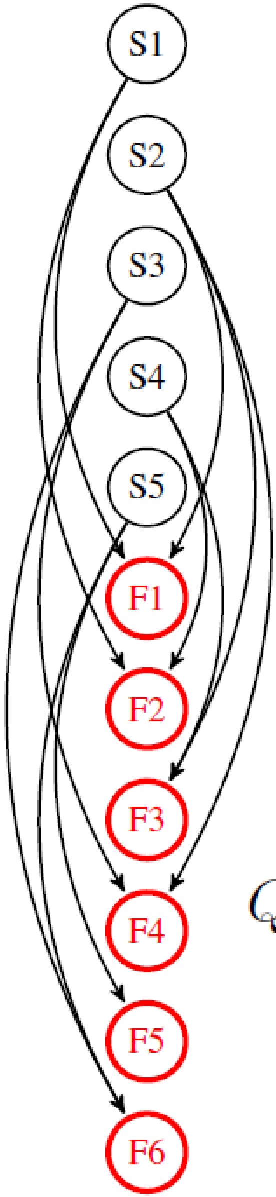
$$C(T_i)\frac{dT_i}{dt} = \sum_j Q_{i,j}$$

Flow components

$$Q_i = G_i(T_{i,a} - T_{i,b})$$

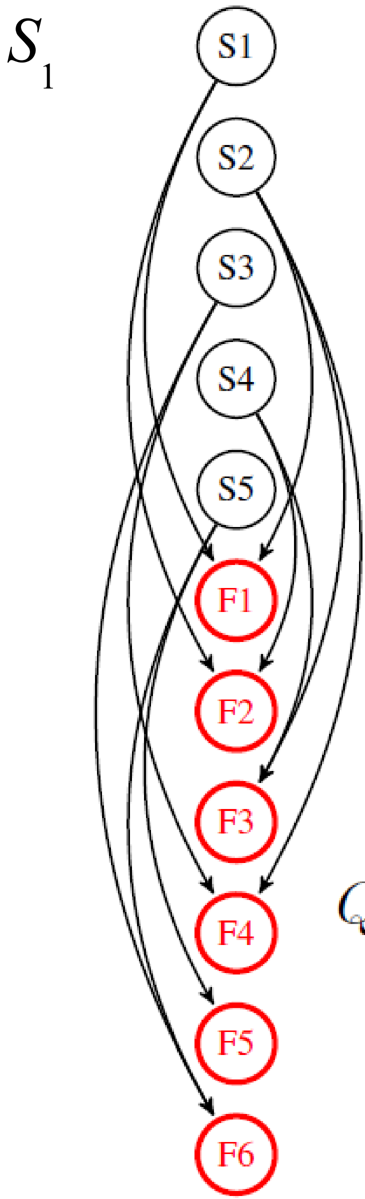
Example 1: Application of the algorithm

S_1



$$Q_i = G_i(T_{i,a} - T_{i,b})$$

Example 1: Application of the algorithm

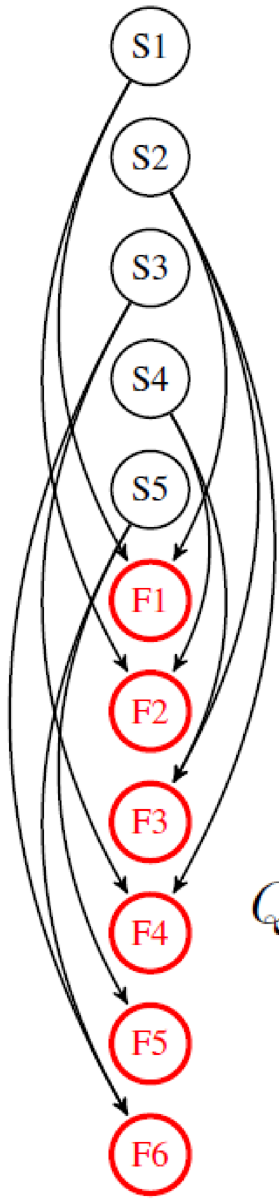


$$C(T_i)\frac{dT_i}{dt} = \sum_j Q_{i,j}$$

$$Q_i = G_i(T_{i,a} - T_{i,b})$$

Example 1: Application of the algorithm

S_1



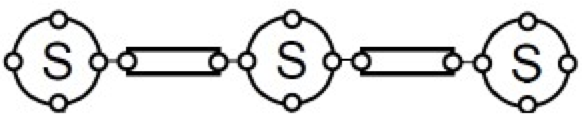
S_2

$$C(T_i)\frac{dT_i}{dt} = \sum_j Q_{i,j}$$

$$Q_i = G_i(T_{i,a} - T_{i,b})$$

Always 2 sets S_i regardless of network dimension!

Example 2: Thermo-Hydraulic Networks



Storage components

$$\begin{bmatrix} e_i & h_i & \rho_i & \frac{\partial \rho_i}{\partial p} & \frac{\partial \rho_i}{\partial T} & \frac{\partial e_i}{\partial p} & \frac{\partial e_i}{\partial T} \end{bmatrix} = f(p_i, T_i)$$

$$M_i = \rho_i V_i$$

$$\frac{dM_i}{dt} = \sum_j w_{i,j}$$

$$\frac{dE_i}{dt} = \sum_j w_{i,j} h_{i,j} + \sum_j Q_{i,j}$$

$$\frac{dM_i}{dt} = \frac{\partial \rho_i}{\partial p}\frac{dp_i}{dt} + \frac{\partial \rho_i}{\partial T}\frac{dT_i}{dt}$$

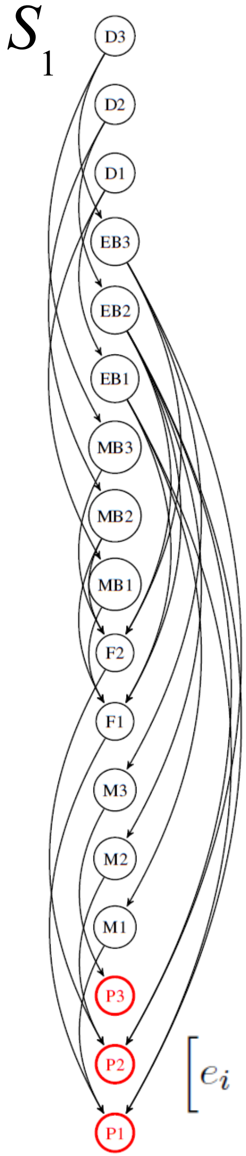
$$\frac{dE_i}{dt} = \left(\frac{\partial e_i}{\partial p}\frac{dp_i}{dt} + \frac{\partial e_i}{\partial T}\frac{dT_i}{dt}\right) M_i + e_i \frac{dM_i}{dt}$$

Flow components

$$Q_i = G_i (T_{i,a} - T_{i,b})$$

$$w_i = w(p_{i,a}, p_{i,b}, \rho_i)$$

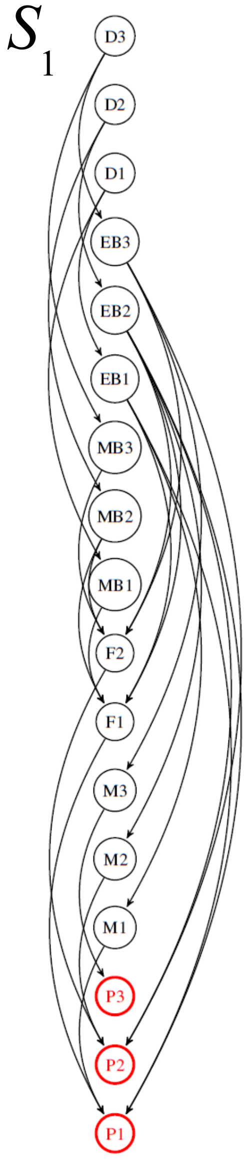
Example 2: Thermo-Hydraulic Networks

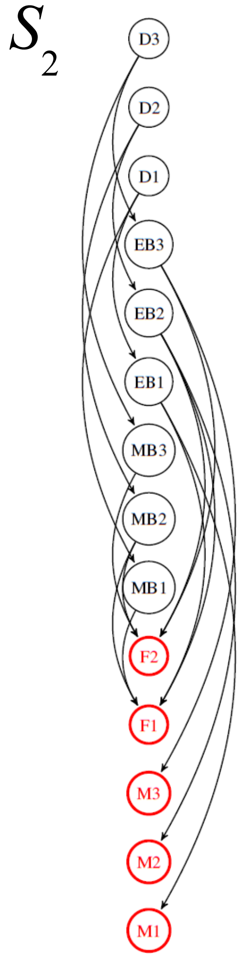


- Often takes up a large fraction of CPU time
- Independent computation for each storage component

$$\begin{bmatrix} e_i & h_i & \rho_i & \frac{\partial \rho_i}{\partial p} & \frac{\partial \rho_i}{\partial T} & \frac{\partial e_i}{\partial p} & \frac{\partial e_i}{\partial T} \end{bmatrix} = f(p_i, T_i)$$

Example 2: Thermo-Hydraulic Networks





$$w_i = w(p_{i,a}, p_{i,b}, \rho_i)$$

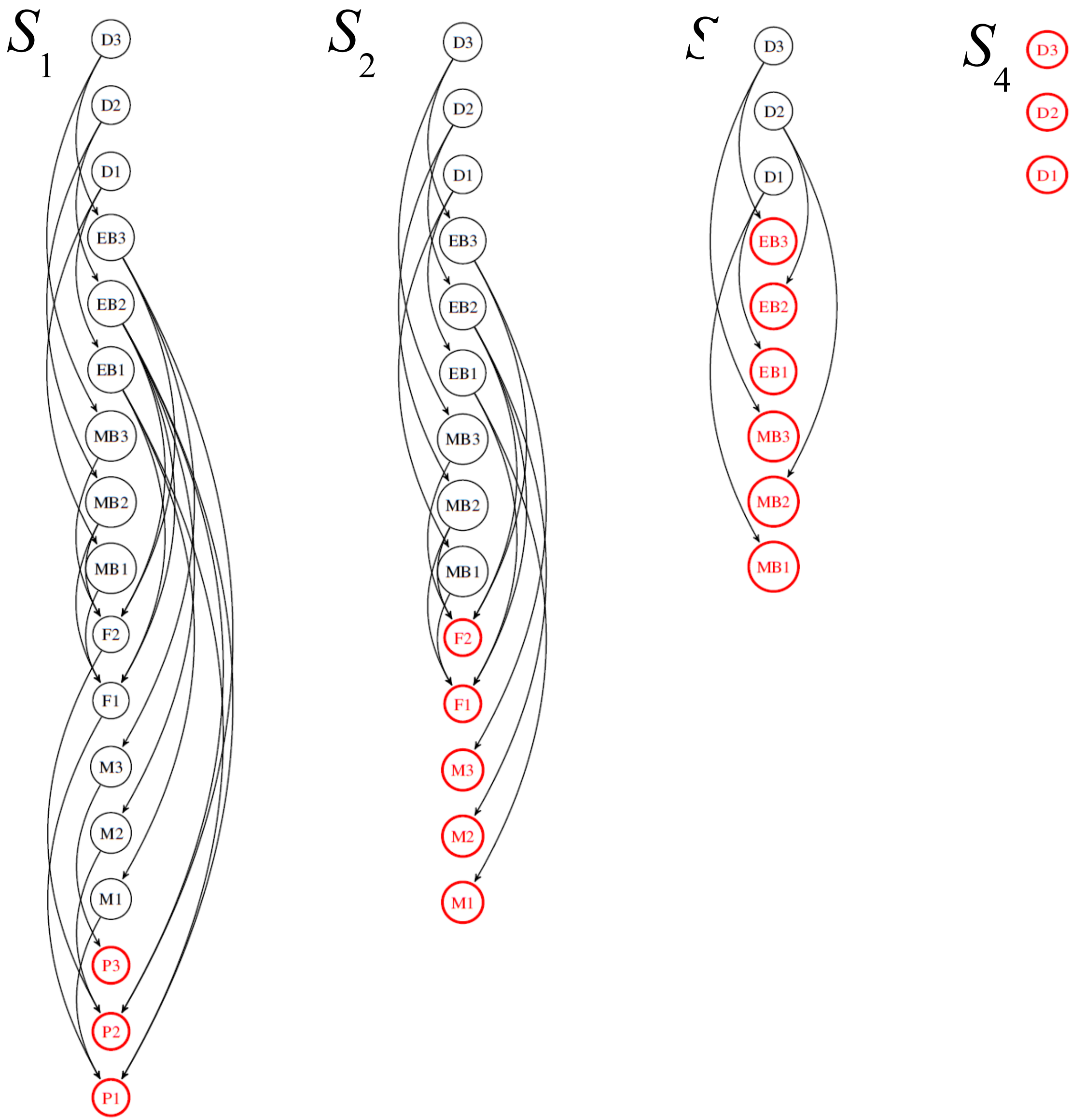
$$M_i = \rho_i V_i$$

Example 2: Thermo-Hydraulic Networks

$$\frac{dE_i}{dt} = \sum_j w_{i,j} h_{i,j} + \sum_j Q_{i,j}$$

$$\frac{dM_i}{dt} = \sum_j w_{i,j}$$

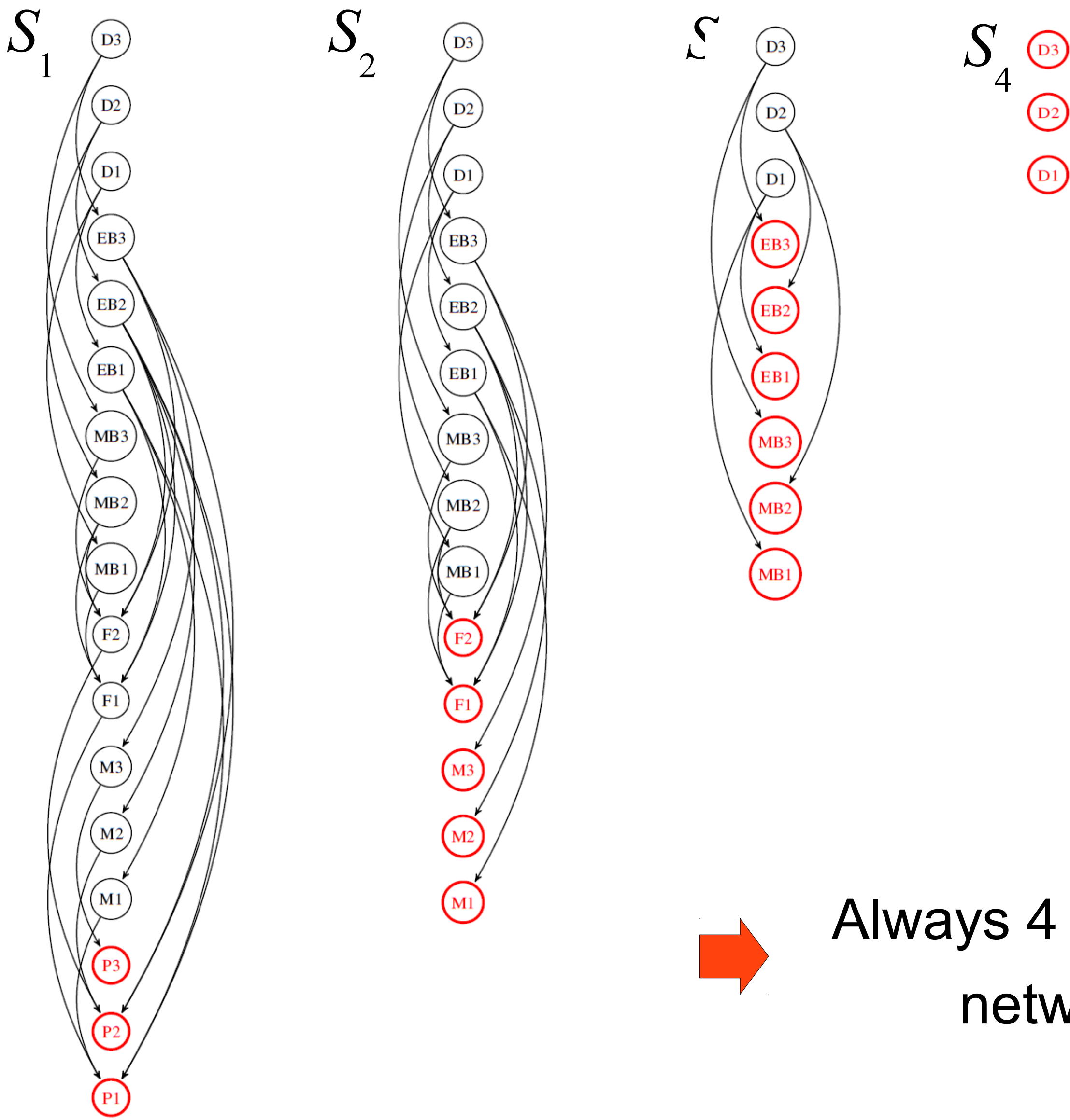
Example 2: Thermo-Hydraulic Networks



$$\frac{dM_i}{dt} = \frac{\partial \rho_i}{\partial p}\frac{dp_i}{dt} + \frac{\partial \rho_i}{\partial T}\frac{dT_i}{dt}$$

$$\frac{dE_i}{dt} = \left(\frac{\partial e_i}{\partial p}\frac{dp_i}{dt} + \frac{\partial e_i}{\partial T}\frac{dT_i}{dt}\right) M_i + e_i \frac{dM_i}{dt}$$

Example 2: Thermo-Hydraulic Networks



$$\frac{dM_i}{dt} = \frac{\partial \rho_i}{\partial p}\frac{dp_i}{dt} + \frac{\partial \rho_i}{\partial T}\frac{dT_i}{dt}$$

$$\frac{dE_i}{dt} = \left(\frac{\partial e_i}{\partial p}\frac{dp_i}{dt} + \frac{\partial e_i}{\partial T}\frac{dT_i}{dt}\right) M_i + e_i \frac{dM_i}{dt}$$

Always 4 sets S_i regardless of network dimension!

Task Scheduling

- Shared memory $\rightarrow$ negligible communication costs
  (beware of L1 caching issues!)

- The larger $\#(S_i)$ / N_{cores} is, the less the actual scheduling policy matters

Task Scheduling

- Shared memory → negligible communication costs
  (beware of L1 caching issues!)

- The larger $\#(S_i)$ / N_{cores} is, the less the actual scheduling policy matters
  - if many more independent tasks than cores, a first-come, first-served scheduling policy will result in all the cores running almost all the time

- Avoid waiting for the slowes one: start slowest tasks earlier
  - minimize chance of N-1 cores waiting for the Nth to complete its task

- Avoid overhead: merge several short tasks in a bigger one

- Only rough estimates of the running time are required

Task Scheduling

- Shared memory $\rightarrow$ negligible communication costs
  (beware of L1 caching issues!)

- The larger $\#(S_i)$ / N_{cores} is, the less the actual scheduling policy matters
  - if many more independent tasks than cores, a first-come, first-served scheduling policy will result in all the cores running almost all the time

- Avoid waiting for the slowes one: start slowest tasks earlier
  - minimize chance of N-1 cores waiting for the Nth to complete its task

- Avoid overhead: merge several short tasks in a bigger one

- Only rough estimates of the running time are required

- Set up threads at the beginning of the simulation,
  activate them @ each time step

Conclusions

- Exploiting Moore's law from 2008 requires exploiting parallelism
- Strategies for parallel simulation of O-O declarative models exist, not implemented in mainstream tools yet (implementation difficulties, not enough cores to justify the overhead)
- A very simple algorithm has been proposed in this work
- It can provide nearly optimal parallel allocation of resources in generalized network models
- Only very rough estimates of task execution times are needed
- Other parallelization strategies could be implemented together with this one to improve overall performance
  - parallel numerical computation of Jacobians
  - parallel solution of large implicit systems in stiff solvers
  - ...
- Beware of thread switching overhead and L1 caching issues!

Thank you for you kind attention!