Import of distributed parameter models into lumped parameter model libraries for the example of linearly deformable solid bodies

3rd International Workshop on Equation-Based Object-Oriented Modeling Languages and Tools

Oslo, Norway, October 3, 2010

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Import of distributed parameter models into lumped parameter model libraries for the example of linearly deformable solid bodies

1. Introduction
2. General Considerations
3. Import of Flexible Bodies
4. Simulation Examples
5. Summary and Outlook

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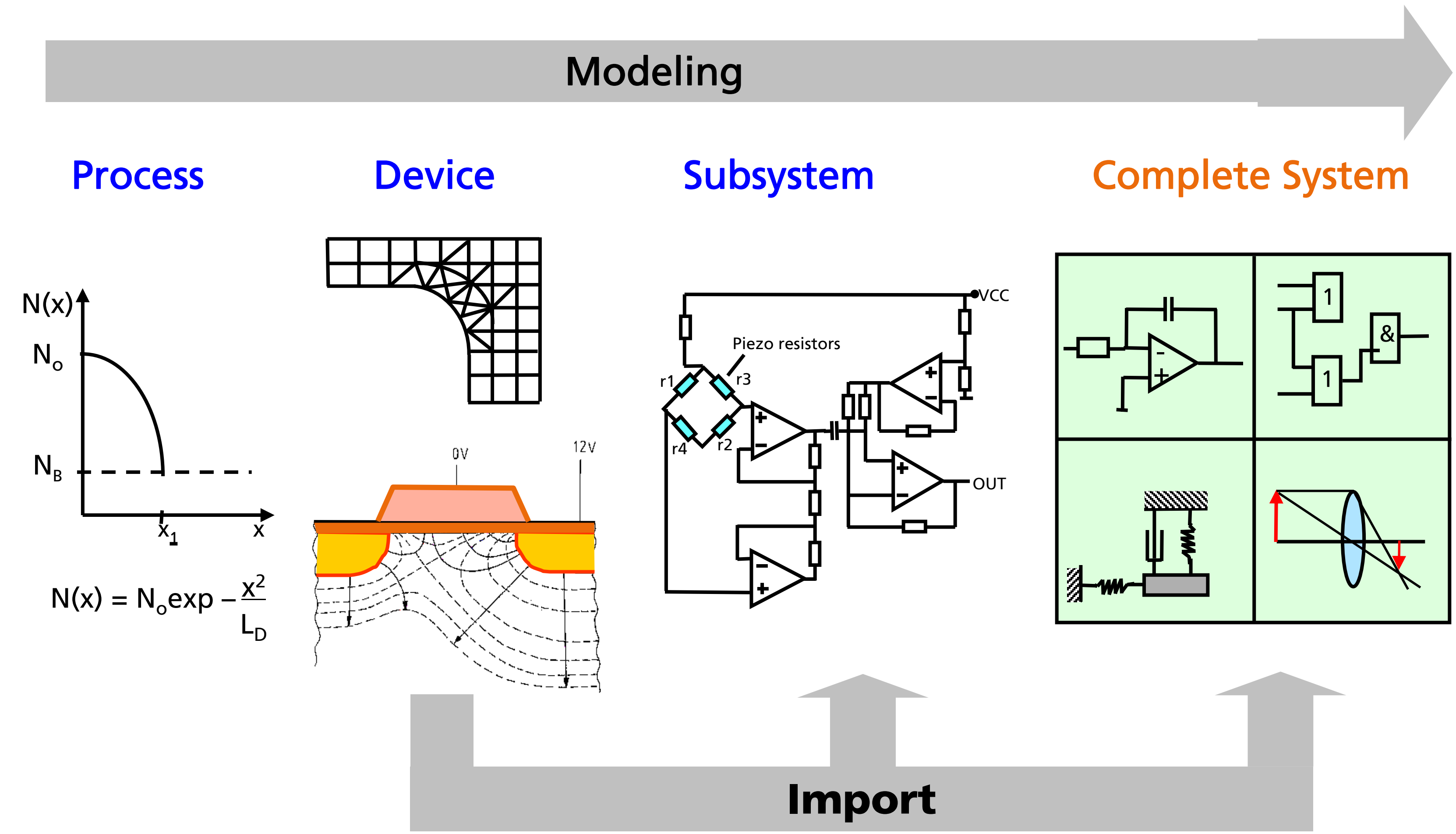
Introduction
Simulation of Heterogeneous Systems



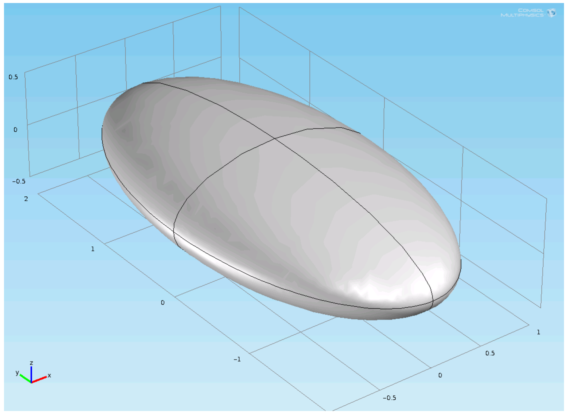
Models

Model: Distributed parameter model

$$\mathcal{D}_t \boldsymbol{u} + \mathcal{R}\boldsymbol{u} = \boldsymbol{w}$$

with

- appropriate linear differential operators $\mathcal{D}_t$ and $\mathcal{R}$ of maximal second order wrt. time
- and suitable initial and boundary conditions



Idea:

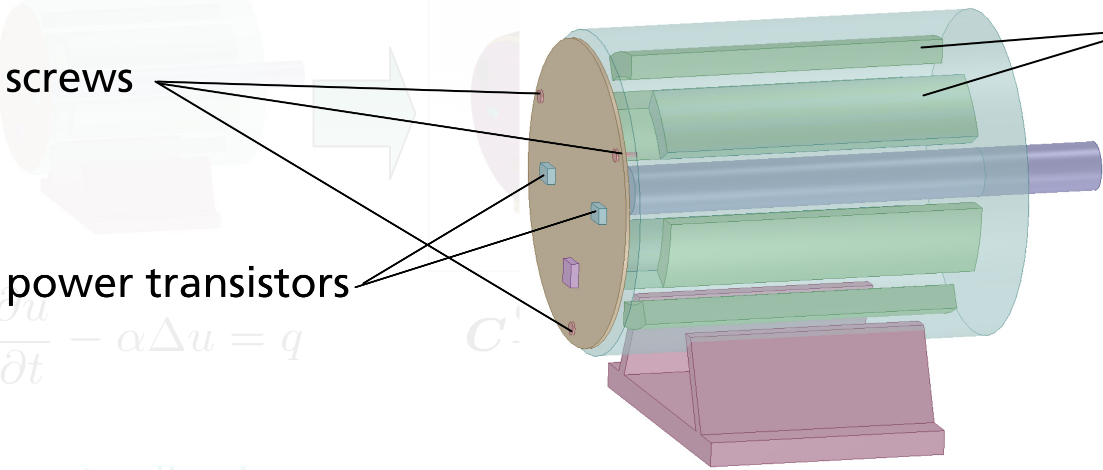
- Import of distributed parameter systems into lumped parameter libraries

Issues:

- Simulation can only be done on a finite dimensional model ⇨ discretization
- We need interaction with other models ⇨ connector definition
- Models tend to be very complex and large in scale ⇨ model order reduction

Introductory Example

Motor:

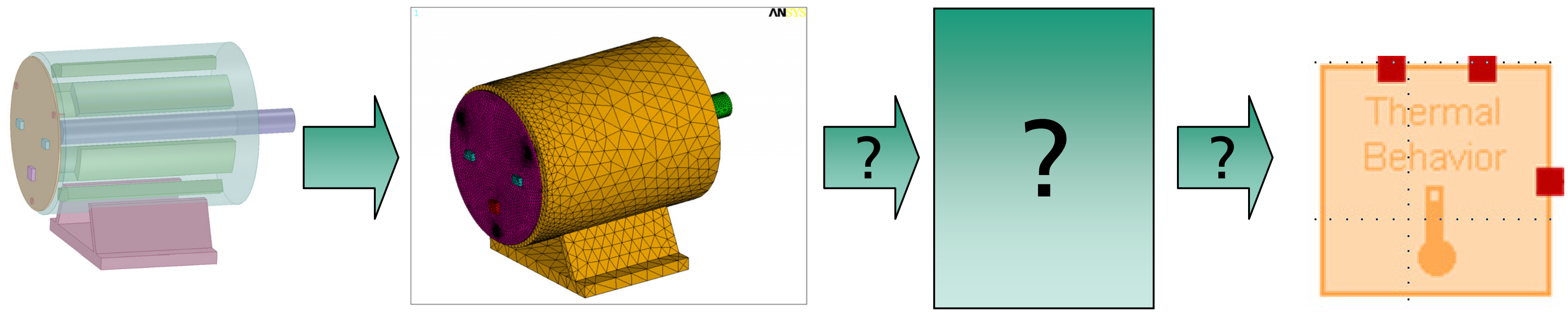


Objective: Thermal model of the motor

Introductory Example

Heat Flow in a Motor

Motor | Discretized | Connectors | Modelica Model



$$\frac{\partial u}{\partial t} - \alpha \Delta u = q \qquad \boldsymbol{C}\frac{d\boldsymbol{\xi}}{dt} - \boldsymbol{G}\boldsymbol{\xi} = \boldsymbol{\eta} \qquad \mathcal{C} = \{\vartheta, \dot{Q}\}$$

???

Other Applications:

- Electrical Transmission Line
- Structural Mechanics

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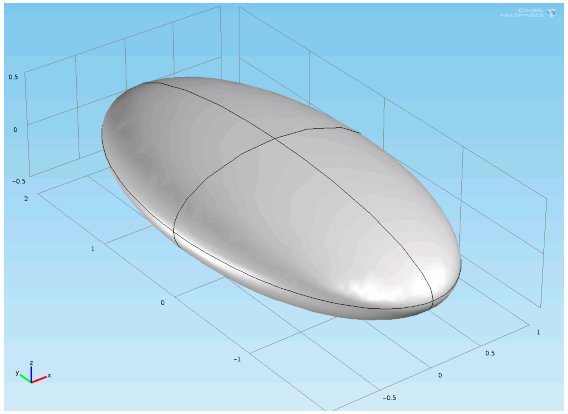
General Considerations
Discretization



Model:

$$\mathcal{D}_t \boldsymbol{u} + \mathcal{R}\boldsymbol{u} = \boldsymbol{w}$$

with

- appropriate linear differential operators $\mathcal{D}_t$ and $\mathcal{R}$ of maximal second order with wrt. time
- and suitable initial and boundary conditions

Discretization:

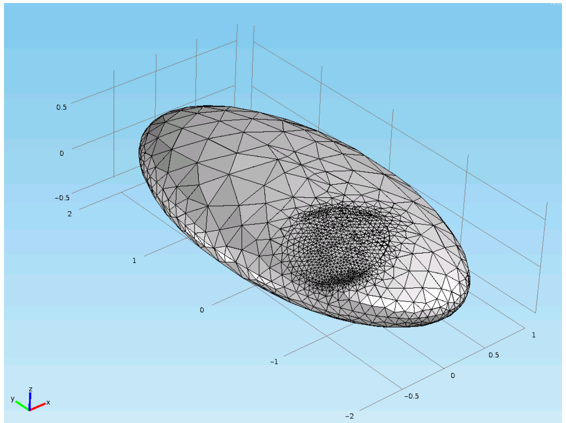
- Simulation can only be done on a finite dimensional model => discretization
- Different Methods:
  - FEM (Finite Element method)
  - FDM (Finite Difference method)
  - FVM (Finite Volume method)
  - BEM (Boundary Element method)

General Considerations
Discretization

Discretized model:

$$\boldsymbol{A}_2 \frac{d^2\boldsymbol{\xi}}{dt^2} + \boldsymbol{A}_1 \frac{d\boldsymbol{\xi}}{dt} + \boldsymbol{A}_0\boldsymbol{\xi} = \boldsymbol{\eta}$$

with appropriate initial conditions



Variables:

- Variables $\boldsymbol{u}$ collected in $\boldsymbol{\xi}^{\mathrm{T}}(t) \equiv \begin{pmatrix} \boldsymbol{u}^{\mathrm{T}}(\boldsymbol{R}_1, t) & \boldsymbol{u}^{\mathrm{T}}(\boldsymbol{R}_2, t) & \dots & \boldsymbol{u}^{\mathrm{T}}(\boldsymbol{R}_N, t) \end{pmatrix}$
- Element Shape integrals of variables $\boldsymbol{w}$ collected in $\boldsymbol{\eta}(t)$

- Mechanics: Mass matrix $\boldsymbol{A}_2$, damping matrix $\boldsymbol{A}_1$, and stiffness matrix $\boldsymbol{A}_0$
- Heat flow: Heat capacitance matrix $\boldsymbol{A}_1$, heat conductance matrix $-\boldsymbol{A}_0$

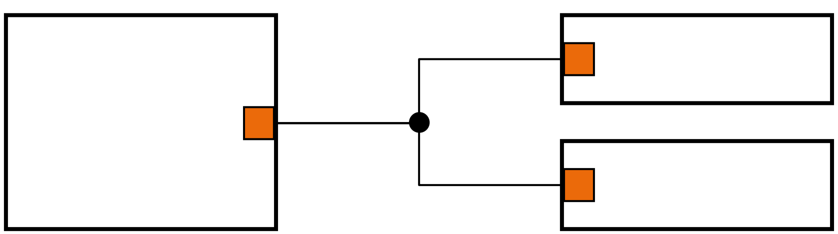
General Considerations
Connector Definitions

Idea:

- Definition of an appropriate interface to the lumped parameter simulation library

Connectors:

- Element that enables the defined interconnection between subsystems
- Enables interaction, modularity and exchangeability



General Considerations
Connector Definitions

Connectors of the lumped parameter model:

- (finite) set $\mathcal{C}$ of lumped connector variables
- In which some elements can be combined to vectors $\boldsymbol{\xi}^m$ and $\boldsymbol{\eta}^m$ (across and through variables)

Heat flow

$$\boldsymbol{\xi}^m = \vartheta$$

$$\boldsymbol{\eta}^m = \dot{Q}$$

Connector Definitions

Connectors of the lumped parameter model:

- (finite) set $\mathcal{C}$ of lumped connector variables
- In which some elements can be combined to vectors $\boldsymbol{\xi}^m$ and $\boldsymbol{\eta}^m$ (across and through variables)

Mechanics

$$\boldsymbol{\xi}^m = \begin{pmatrix} \boldsymbol{u}^{\mathrm{T}} & \boldsymbol{\vartheta}^{\mathrm{T}} \end{pmatrix}^{\mathrm{T}}$$
$$\boldsymbol{\eta}^m = \begin{pmatrix} \boldsymbol{f}^{\mathrm{T}} & \boldsymbol{t}^{\mathrm{T}} \end{pmatrix}^{\mathrm{T}}$$

General Considerations
Connector Definitions

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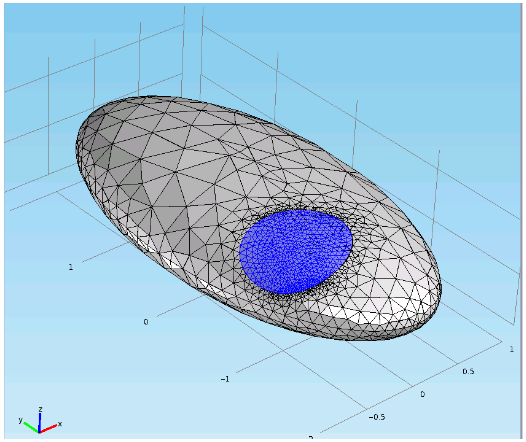
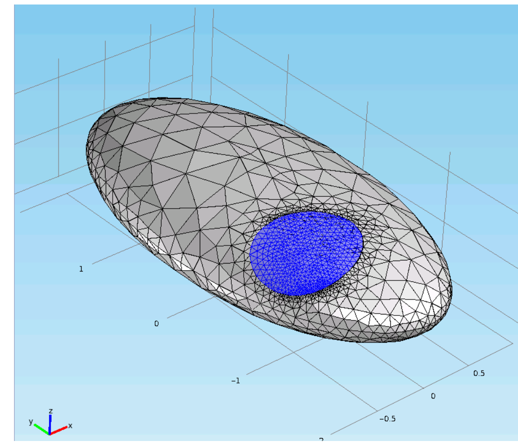
$$\boldsymbol{\eta}^m = \begin{pmatrix} \boldsymbol{f}^{\mathrm{T}} & \boldsymbol{t}^{\mathrm{T}} \end{pmatrix}^{\mathrm{T}}$$

Connectors of the discretized model:

- Set $\mathcal{C}$ (vectors $\boldsymbol{\xi}^m$ and $\boldsymbol{\eta}^m$)
- vectors $\boldsymbol{\xi}^s$ and $\boldsymbol{\eta}^s$ being the vectors of all (dependent) variables of those nodes that lie within
- property:
  - Vectors $\boldsymbol{\xi}^m$ and $\boldsymbol{\eta}^m$ uniquely describe behavior of variables $\boldsymbol{\xi}^s$ and $\boldsymbol{\eta}^s$
  - "Valid" configurations of $\boldsymbol{\xi}^s$ and $\boldsymbol{\eta}^s$ uniquely determine the variables $\boldsymbol{\xi}^m$ and $\boldsymbol{\eta}^m$



General Considerations
Connector Definitions for the Discretized Model



How to define?

- Behavior of $\boldsymbol{\xi}^s$ and $\boldsymbol{\eta}^s$ must be expressed in terms of $\boldsymbol{\xi}^m$ and $\boldsymbol{\eta}^m$ by two injective functions
- Can be interpreted as constraint equations
- Assuming linear constraint equations:

$$\boldsymbol{\xi}^s = \hat{\boldsymbol{\Phi}}\boldsymbol{\xi}^m \qquad \boldsymbol{\eta}^s = \hat{\boldsymbol{\Psi}}\boldsymbol{\eta}^m$$

- Expressing new by old variables:

$$\boldsymbol{\xi} = \begin{pmatrix} \boldsymbol{\xi}^r \\ \boldsymbol{\xi}^s \end{pmatrix} = \begin{pmatrix} \mathbf{0} & \boldsymbol{I} \\ \hat{\boldsymbol{\Phi}} & \mathbf{0} \end{pmatrix} \begin{pmatrix} \boldsymbol{\xi}^m \\ \boldsymbol{\xi}^r \end{pmatrix} \equiv \bar{\boldsymbol{\Phi}}\bar{\boldsymbol{\xi}}$$

Result:

$$\boxed{\begin{aligned} \bar{\boldsymbol{A}}_2\ddot{\bar{\boldsymbol{\xi}}} + \bar{\boldsymbol{A}}_1\dot{\bar{\boldsymbol{\xi}}} + \bar{\boldsymbol{A}}_0\bar{\boldsymbol{\xi}} &= \bar{\boldsymbol{\eta}}_e + \bar{\boldsymbol{B}}\boldsymbol{\eta}^m \\ \boldsymbol{\xi}^m &= \bar{\boldsymbol{C}}\bar{\boldsymbol{\xi}} \end{aligned}}$$

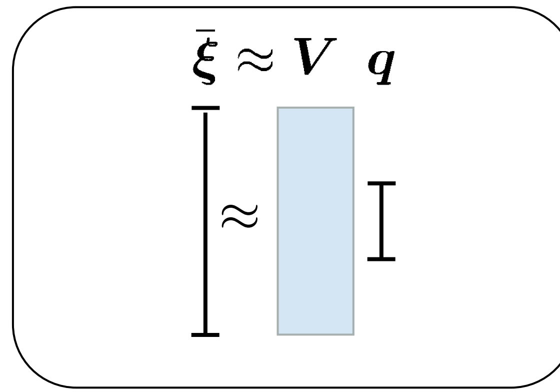
Model Order Reduction

Why do we often need model order reduction?

- Discretized model tend to get large in scale
- Equation-based simulation tools are not able to handle large-scale dynamical systems

Objective?

- Approximation of the model behavior by a model of lower dimension with the same structure as the original model



How do we apply that?

- Considering as constraint equation on allowed solutions
- Using Lagrange Multiplier Theorem

$$\boldsymbol{A}_{q,2}\ddot{\boldsymbol{q}} + \boldsymbol{A}_{q,1}\dot{\boldsymbol{q}} + \boldsymbol{A}_{q,0}\boldsymbol{q} = \boldsymbol{\eta}_{e,q} + \boldsymbol{B}_q\boldsymbol{\eta}^m$$
$$\boldsymbol{\xi}^m = \boldsymbol{C}_q\boldsymbol{q}$$

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3. Import of Flexible Bodies
   - Connector Definitions
   - Discretization
   - Model Order Reduction
   - Nonlinear Equations
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Import of Flexible Bodies
Task

Task:

- Including dynamic behavior of deformable bodies into classical multi-body libraries

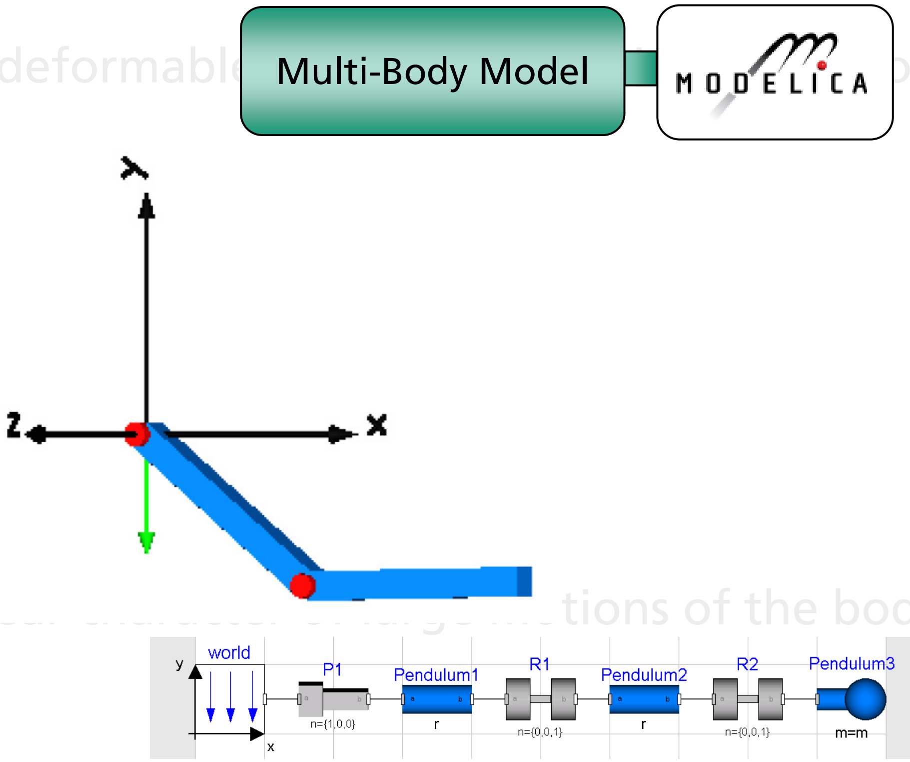
Import of Flexible Bodies
Task

Task:

- Including dynamic behavior of deformabl… …ody libraries

Challenges:

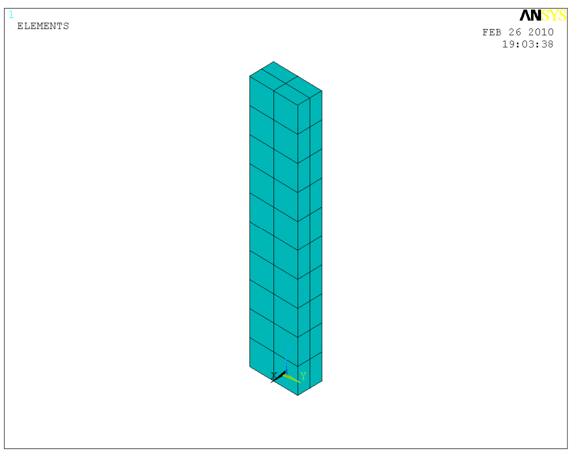
- Discretization
- Connector Definition
- Model order reduction
- Taking into account the nonli… …tions of the body

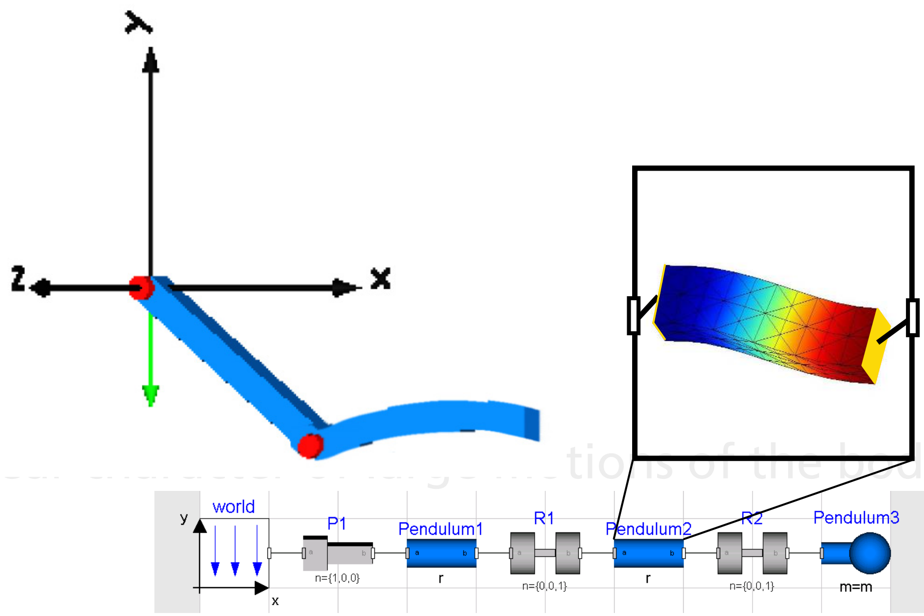


Import of Flexible Bodies
Task

Finite Element Model

Multi-Body Model





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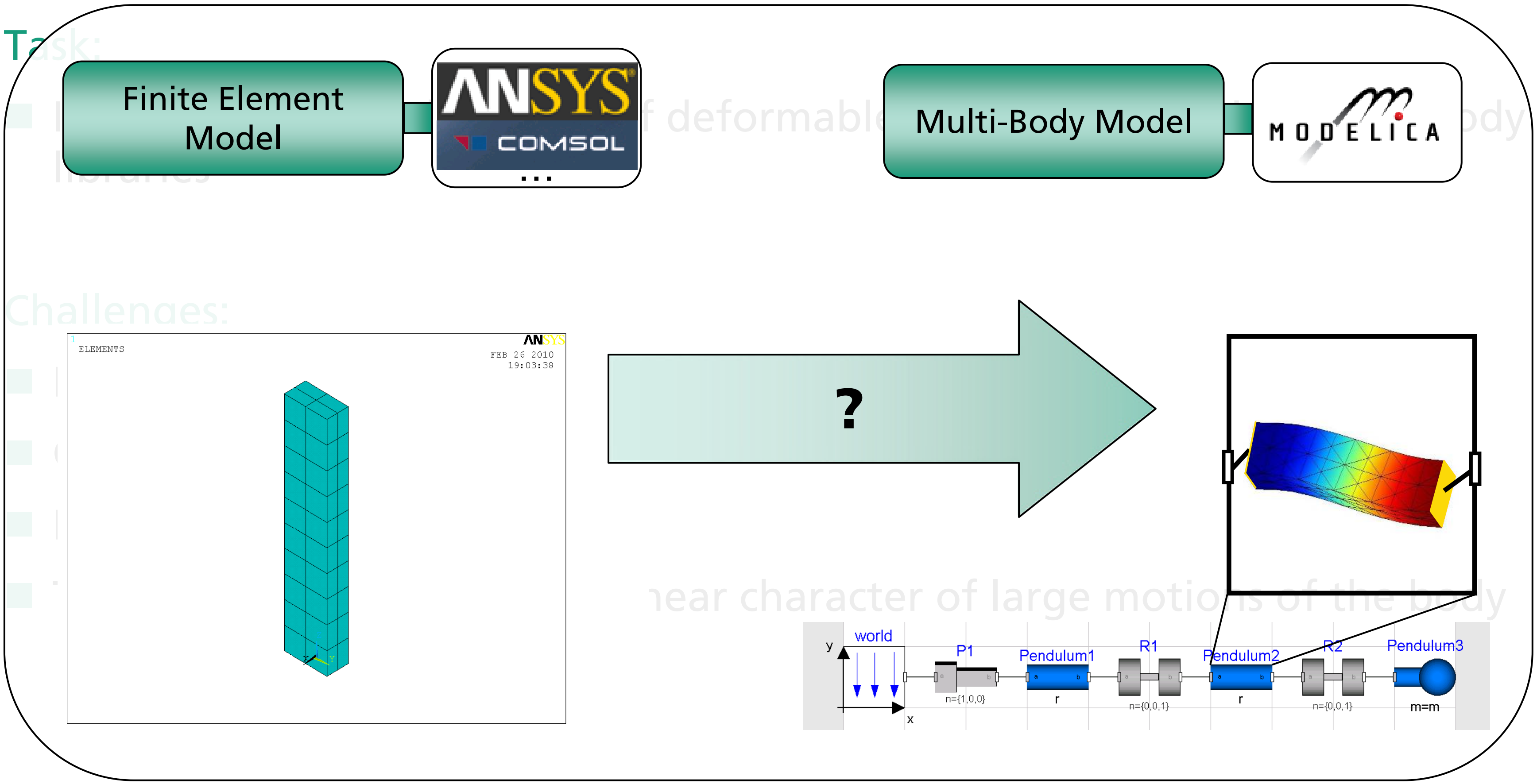
Import of Flexible Bodies
Task



Import of Flexible Bodies
Task

Task:

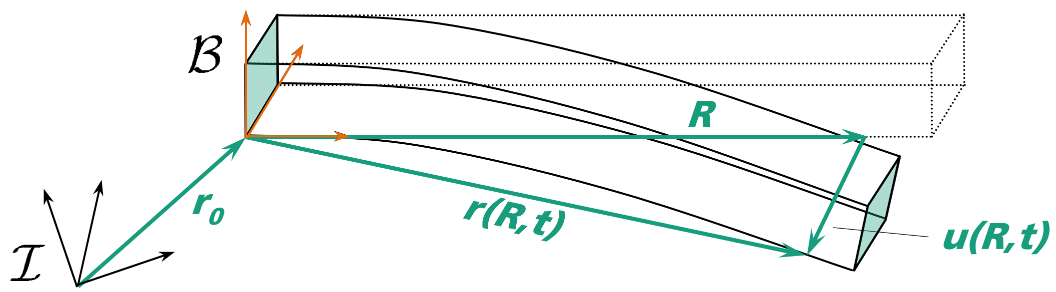
- Including dynamic behavior of deformable bodies into classical multi-body libraries

Challenges:

- Discretization
- Connector Definition
- Model order reduction
- Taking into account the nonlinear character of large motions of the body

Import of Flexible Bodies
Idea

Sketch:



Idea:

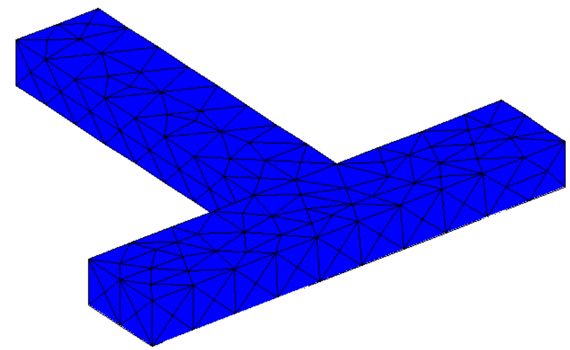
- Considering the motion as a superposition of
  - Motion of reference frame $\mathcal{B}$
  - Small deformations of body
- Describing of deformation through linear Finite Element Model
- Considering every connector as a rigid body
- Fixing reference frame to one connector

Import of Flexible Bodies
Discretization

Discretized model:

$$\boldsymbol{M}\frac{d^2\boldsymbol{\xi}}{dt^2} + \boldsymbol{D}\frac{d\boldsymbol{\xi}}{dt} + \boldsymbol{K}\boldsymbol{\xi} = \boldsymbol{\eta}$$

with appropriate initial conditions



Variables:

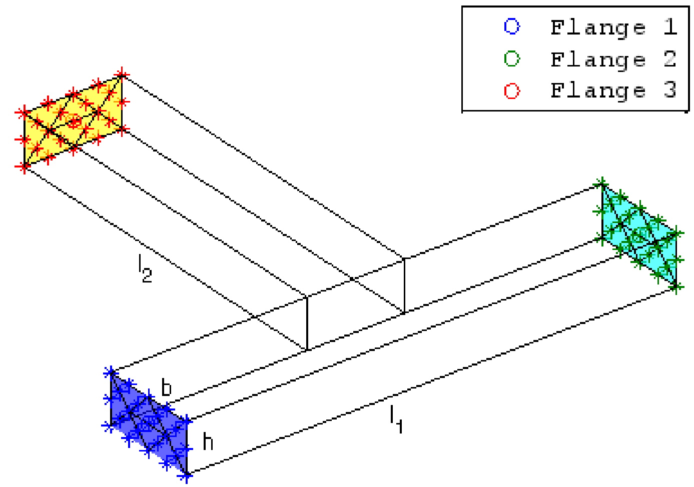
- Displacements $\boldsymbol{u}$ collected in $\boldsymbol{\xi}^{\mathrm{T}} \equiv \big(\boldsymbol{u}^{\mathrm{T}}(\boldsymbol{R}_1) \quad \boldsymbol{u}^{\mathrm{T}}(\boldsymbol{R}_2) \quad \ldots \quad \boldsymbol{u}^{\mathrm{T}}(\boldsymbol{R}_N)\big)$
- Forces $\boldsymbol{f}$ collected in $\boldsymbol{\eta}^{\mathrm{T}} \equiv \big(\boldsymbol{f}^{\mathrm{T}}(\boldsymbol{R}_1) \quad \boldsymbol{f}^{\mathrm{T}}(\boldsymbol{R}_2) \quad \ldots \quad \boldsymbol{f}^{\mathrm{T}}(\boldsymbol{R}_N)\big)$
- Mass matrix $\boldsymbol{M}$, damping matrix $\boldsymbol{D}$, and stiffness matrix $\boldsymbol{K}$

Import of Flexible Bodies

Connector Definitions for the Mechanical Model

Connectors:

- "slave nodes": all nodes that should be used to create a connector
- Variables in $\boldsymbol{\xi}^m$ can be seen as the transl. and rot. deflection of a virtual "master node"
- Variables in $\boldsymbol{\eta}^m$ can be interpreted as the forces and torques acting on the virtual "master node"
- slave nodes rigidly attached to master node
- coordinates of master node uniquely determine coordinates of all slave nodes



Constraint equations:

- Assuming small deformations
- Use of geometric linearization

Equations of Motion:

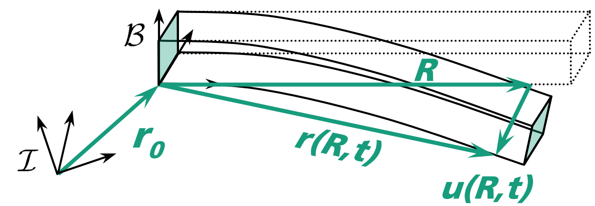
$$\boxed{\bar{\boldsymbol{M}}\ddot{\bar{\boldsymbol{\xi}}} + \bar{\boldsymbol{D}}\dot{\bar{\boldsymbol{\xi}}} + \bar{\boldsymbol{K}}\bar{\boldsymbol{\xi}} = \bar{\boldsymbol{\eta}}_e + \bar{\boldsymbol{B}}\boldsymbol{\eta}^m}$$

$$\boxed{\boldsymbol{\xi}^m = \bar{\boldsymbol{C}}\bar{\boldsymbol{\xi}}}$$

Import of Flexible Bodies
Inclusion of non-linear terms

Idea: Moving reference frame

$$\bar{\boldsymbol{M}}\left(\mathring{\mathring{\bar{\boldsymbol{\xi}}}} + \boxed{\boldsymbol{a}_0}\right) + \bar{\boldsymbol{D}}\mathring{\bar{\boldsymbol{\xi}}} + \bar{\boldsymbol{K}}\bar{\boldsymbol{\xi}} = \bar{\boldsymbol{\Phi}}^{\mathrm{T}}\boldsymbol{\eta}_e + \boxed{\boldsymbol{\eta}_c} + \bar{\boldsymbol{B}}\boldsymbol{\eta}^m$$
$$\boldsymbol{\xi}^m = \bar{\boldsymbol{B}}^{\mathrm{T}}\bar{\boldsymbol{\xi}}$$



Nonlinear terms:

- Acceleration of body frame:

$$\boxed{(\boldsymbol{a}_0)_i} = \begin{cases} \begin{pmatrix} \ddot{\boldsymbol{r}}_0 + (\dot{\tilde{\boldsymbol{\omega}}} + \tilde{\boldsymbol{\omega}}^2)(\tilde{\boldsymbol{R}}_{c,i} + \boldsymbol{\xi}_i) + 2\tilde{\boldsymbol{\omega}}\dot{\boldsymbol{\xi}}_i \\ \dot{\boldsymbol{\omega}} + \tilde{\boldsymbol{\omega}}\dot{\boldsymbol{\xi}}_i \end{pmatrix} & i \in \Omega_m \\ \ddot{\boldsymbol{r}}_0 + (\dot{\tilde{\boldsymbol{\omega}}} + \tilde{\boldsymbol{\omega}}^2)(\tilde{\boldsymbol{R}}_i + \boldsymbol{\xi}_i) + 2\tilde{\boldsymbol{\omega}}\dot{\boldsymbol{\xi}}_i & i \in \Omega_r \end{cases}$$

- Coriolis forces on connectors:

$$\boxed{(\boldsymbol{\eta}_c)_i} = \begin{cases} \begin{pmatrix} \tilde{\boldsymbol{\omega}} & \boldsymbol{0} \\ \boldsymbol{0} & \tilde{\boldsymbol{\omega}} \end{pmatrix} \bar{\boldsymbol{M}}_{i,i} \begin{pmatrix} \boldsymbol{0} \\ \boldsymbol{\omega} \end{pmatrix} & \text{for } i \in \Omega_m \\ \boldsymbol{0} & \text{for } i \in \Omega_r. \end{cases}$$

Import of Flexible Bodies
Model Order Reduction and Fixing Reference Frame

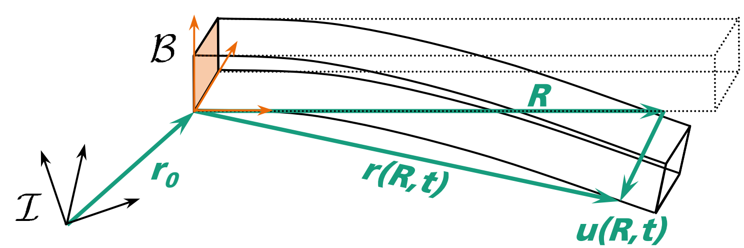
Model Order Reduction:

- Approximation by a model of lower dimension: $\bar{\xi} \approx \boldsymbol{V}\boldsymbol{q}$

Note: Reduction matrix $\boldsymbol{V}$ must include rigid body modes

Result: model of lower dimension but with the same structure as before

Fixing Reference Frame:



- Reference frame has to be fixed to one connector
- Position of other connectors can be expressed in terms of the deformation coordinates and the reference frame position and orientation

Inclusion of non-linear terms

Equations of motion: Moving reference frame

$$\begin{pmatrix} \boxed{\begin{matrix} m\boldsymbol{I} & m\tilde{\boldsymbol{r}}_s^{\mathrm{T}} \\ m\tilde{\boldsymbol{r}}_s & \boldsymbol{\Theta}_0 \end{matrix}} & \boxed{\begin{matrix} \boldsymbol{M}_{b1}^{\mathrm{T}} \\ \boldsymbol{M}_{b2}^{\mathrm{T}} \end{matrix}} \\ \boxed{\begin{matrix} \boldsymbol{M}_{b1} & \boldsymbol{M}_{b2} \end{matrix}} & \boxed{\boldsymbol{M}_{\bar{q}}} \end{pmatrix} \begin{pmatrix} \boxed{\begin{matrix} \ddot{\boldsymbol{r}}_0 \\ \dot{\boldsymbol{\omega}} \end{matrix}} \\ \boxed{\ddot{\bar{\boldsymbol{q}}}} \end{pmatrix} + \boxed{\check{\boldsymbol{D}}\dot{\bar{\boldsymbol{q}}} + \check{\boldsymbol{K}}\bar{\boldsymbol{q}}} + \boxed{\boldsymbol{g}(\boldsymbol{\omega}, \bar{\boldsymbol{q}}, \dot{\bar{\boldsymbol{q}}})} = \boldsymbol{V}^{\mathrm{T}}\bar{\boldsymbol{\Phi}}^{\mathrm{T}}\boldsymbol{\eta}_e + \boxed{\check{\boldsymbol{B}}\boldsymbol{\eta}^m}$$

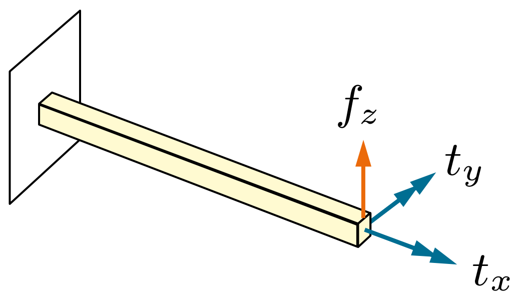
$$\boxed{\boldsymbol{\xi}^m = \boldsymbol{B}_{\bar{q}}^{\mathrm{T}}\bar{\boldsymbol{q}}}$$

- Coupling matrices: $\boldsymbol{M}_{b1}, \boldsymbol{M}_{b2}$
- Nonlinear terms: $\boldsymbol{g}(\boldsymbol{\omega}, \bar{\boldsymbol{q}}, \dot{\bar{\boldsymbol{q}}})$

Import of distributed parameter models into lumped parameter model libraries for the example of linearly deformable solid bodies

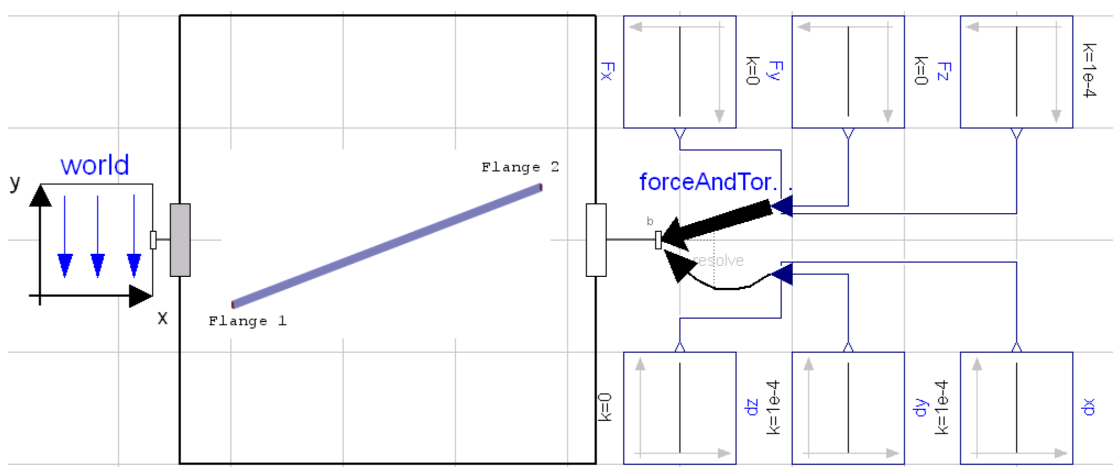
Static Deformation of a long Beam

Experiment Setup:



$$f_z = 1 \cdot 10^{-4}\,\mathrm{N}$$
$$t_x = 1 \cdot 10^{-4}\,\mathrm{Nm}$$
$$t_y = 1 \cdot 10^{-4}\,\mathrm{Nm}$$
$$E = 2 \cdot 10^{11}\,\mathrm{Pa}$$

Dymola Model:



Simulation Examples

Static Deformation of a long Beam

Relative Error:

	Theory	ANSYS
u_z	0,06 %	$4.08 \cdot 10^{-3}$ %
φ_x	0,80 %	$1.75 \cdot 10^{-3}$ %
φ_y	0,08 %	$1.36 \cdot 10^{-3}$ %

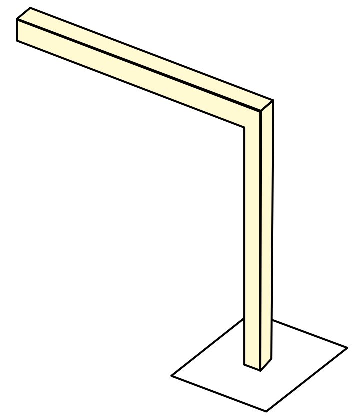
Interpretation:

Relative errors between the simulation results from Dymola and

- the ANSYS results as well as
- the theoretic results

are sufficiently small

Simulation Examples
Eigenfrequency Analysis of an L-shaped Beam



Experiment Setup:

- no damping

$$h = 5\,\mathrm{mm} \qquad E = 2.1 \cdot 10^{11}\,\mathrm{Pa}$$

$$b = 12\,\mathrm{mm} \qquad G = 82 \cdot 10^{9}\,\mathrm{Pa}$$

$$l = 1\,\mathrm{m} \qquad \rho = 7900\,{}^{\mathrm{kg}}/_{\mathrm{m}^3}$$

Results:

	f_1	f_2	f_3	f_4	f_5
Theory	3.331 Hz	9.070 Hz	44.772 Hz	66.687 Hz	143.179 Hz
ANSYS	3.337 Hz	9.121 Hz	44.802 Hz	66.030 Hz	143.173 Hz
Dymola	3.337 Hz	9.121 Hz	44.802 Hz	66.030 Hz	143.173 Hz

Simulation Examples
Eigenfrequency Analysis of an L-shaped Beam

Eigen modes:

Simulation Examples

Eigenfrequency Analysis of an L-shaped Beam

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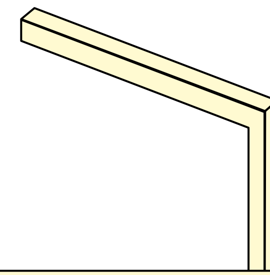

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Relative errors between the simulation results from Dymola and

- the ANSYS results as well as
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Theory	3.331 Hz	9.070 Hz	44.772 Hz	66.687 Hz	143.179 Hz
ANSYS	3.337 Hz	9.121 Hz	44.802 Hz	66.030 Hz	143.173 Hz
Dymola	3.337 Hz	9.121 Hz	44.802 Hz	66.030 Hz	143.173 Hz

Simulation Examples
T-Square under Uniform Rotation

Experiment Setup:

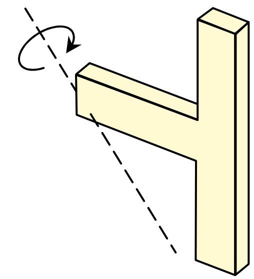
- Rayleigh damping

$b = 4\,\mathrm{cm}$ $\quad$ $E = 2 \cdot 10^{11}\,\mathrm{Pa}$

$h = 2\,\mathrm{cm}$ $\quad$ $\nu = 0.3$

$l_1 = 15\,\mathrm{cm}$ $\quad$ $\rho = 7850\,{}^{\mathrm{kg}}/_{\mathrm{m}^3}$

$l_2 = 15\,\mathrm{cm}$



Results in SimulationX:

	Connector 1	Connector 2
u_x	$9.696 \cdot 10^{-6}$ mm	$5.004 \cdot 10^{-4}$ mm
u_y	$-4.042 \cdot 10^{-4}$ mm	$-1.358 \cdot 10^{-4}$ mm
u_z	$-2.382 \cdot 10^{-3}$ mm	$-3.699 \cdot 10^{-3}$ mm

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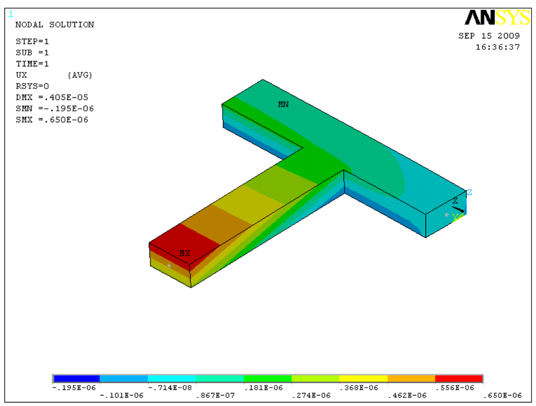
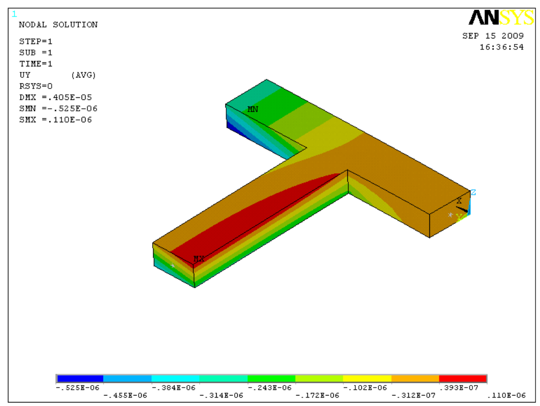
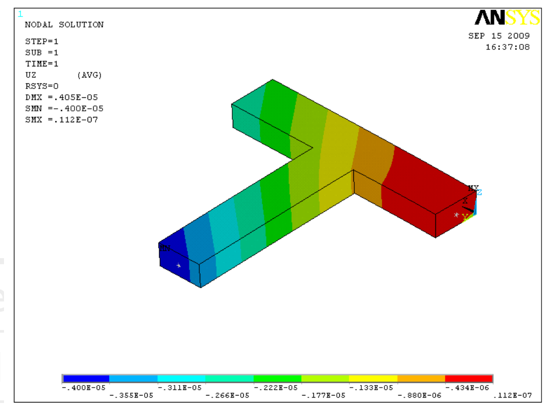
Simulation Examples
T-Square under Uniform Rotation

Experiment Setup:

ANSYS results:







Simulation Examples
Static Deformation of a long Beam

Relative error:

	Connector 1	Connector 2
u_x	0.02 %	0.02 %
u_y	0.02 %	0.02 %
u_z	0.03 %	0.02 %

Interpretation:

Relative errors between the simulation results from SimulationX and ANSYS are sufficiently small

Import of distributed parameter models into lumped parameter model libraries for the example of linearly deformable solid bodies

Summary

Presentation showed

- an approach of including discretized distributed parameter models into libraries of lumped parameter models
- an approach for a connector definition
- the application to flexible bodies
- simulation results to verify the models

Summary and Outlook
Outlook

Further investigations are necessary on

- different connector definitions
- a more general description of the models (port-hamiltonian framework)
- the evaluation of the chosen approach for the import of flexible bodies compared to other approaches
- heterogeneous systems with different physical domains

Import of distributed parameter models into lumped parameter model libraries for the example of linearly deformable solid bodies

Thank you!

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